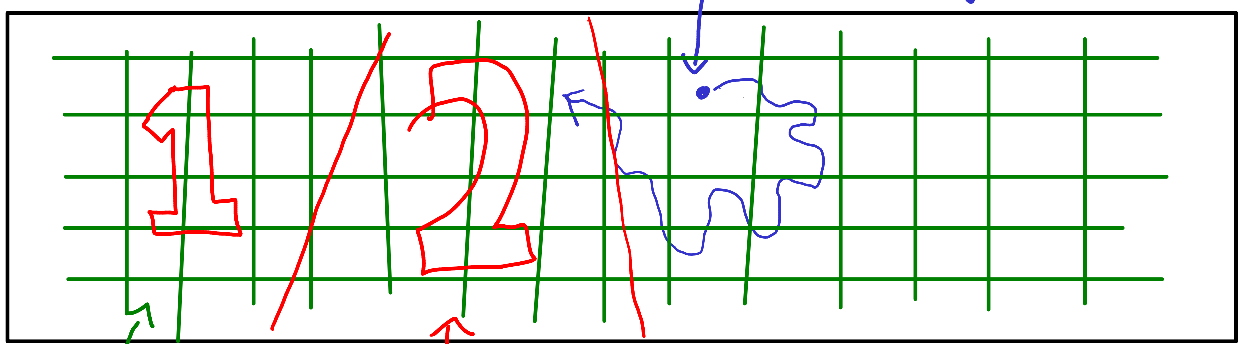
ratchet: states

energies: E_n

total (ratchet + gas): $E_{tot} = E_n + E_n^{env}$

point: config. of ratchet + all gas molec.



phase space represents total (ratchet + gas)

const. E_{tot}

microstate μ of phase vol. a

"macrostate": all pts where system is in state n

ergodicity + mixing: $$p_\mu(t) \xrightarrow[t\to\infty]{} \frac{1}{\Theta(E_{tot})} \equiv \frac{1}{\underbrace{\Theta_{tot}}_{\text{total \# of microstates on surface (\# of cities)}}}$$

$\Theta_n \equiv$ # of microstates in macrostate n (# of cities in country n)

$$\Theta_{tot} = \sum_{i=1}^{N} \Theta_n \qquad N = \text{\# macrostates (\# countries)}$$

assumptions :

- environment has many more degrees of freedom than system

$$\Theta_n >> N$$

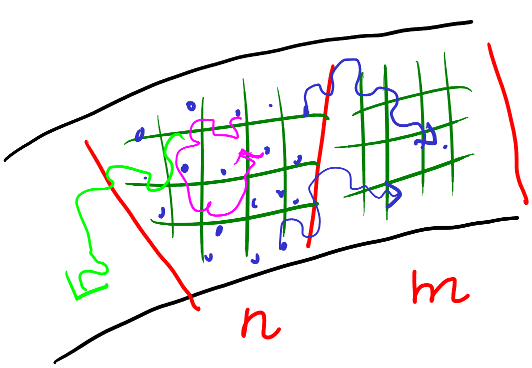
$$\Theta_{tot} >> N$$

- mixing is fast relative to timescales of interest

- dynamics of exploring w/ a macrostate happens faster than transitions b/t macrostates

$\Rightarrow$ under this scenario: probability of visiting next macrostates depends at most on the current one ("memory" of deeper past is lost b/c of rapid randomization due to mixing)

Markov property



define trans. matrix

W_{mn} = prob. to end up in m after time interval δt given start in n

$$= \frac{\#\text{ traj. that start in } n \text{ \& end up in } m \text{ after } \delta t}{\#\text{ traj. " " " \& end up anywhere after } \delta t}$$

$p_n(t_i)$ = prob. to be in macrostate n at time n

$\vec{p}(t_{i+1}) = W\,\vec{p}(t_i)$ DTDS master equ.

$$p_\mu(t) \xrightarrow[t\to\infty]{} \frac{1}{\Theta_{tot}} \Rightarrow p_n(t) \xrightarrow[t\to\infty]{} \frac{\Theta_n}{\Theta_{tot}}$$

uniform distrib over all μ

Θ_n: # boxes in n

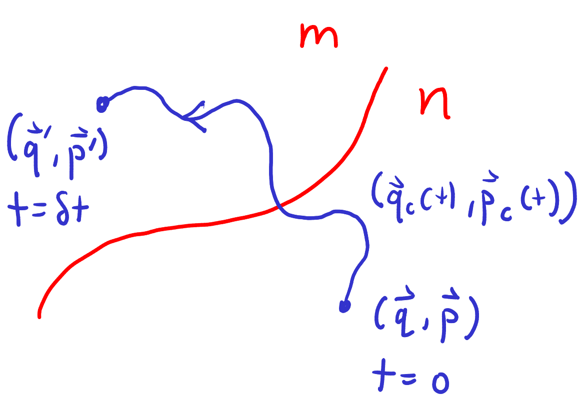
Θ_{tot}: # boxes total

$\equiv p_n^s$

stationary state

$$W\vec{p}^{\,s} = \vec{p}^{\,s}$$

Next step: use time reversal symmetry



classical traj. satisfies:

$$\frac{d\vec{q}_c}{dt} = \frac{\partial H}{\partial \vec{p}_c} \qquad \frac{d\vec{p}_c}{dt} = -\frac{\partial H}{\partial \vec{q}_c}$$

Hamilton equations

"reversed solution" also satisfies these equations:

$$\vec{q}_r(\tilde{t}) \equiv \vec{q}_c(t-\tilde{t})$$
$$\vec{p}_r(\tilde{t}) \equiv -\vec{p}_c(t-\tilde{t})$$