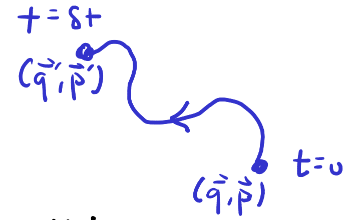
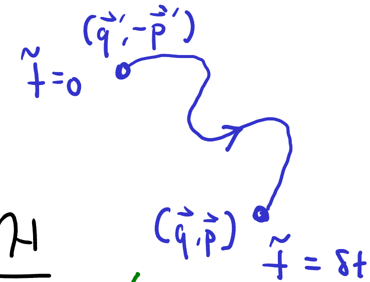
Classical sol'n: $\vec{q}_c(t)$, $\vec{p}_c(t)$



reversed sol'n is also a valid sol'n to Hamilton's eqn

$$\vec{q}_r(\tilde{t}) \equiv \vec{q}_c(t-\tilde{t})$$

$$\vec{p}_r(\tilde{t}) \equiv -\vec{p}_c(t-\tilde{t})$$



proof:

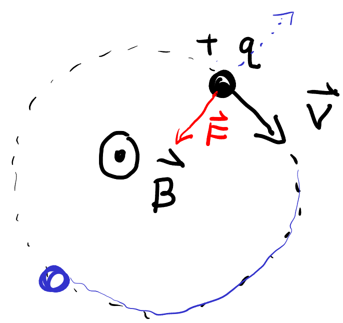
$$\frac{d\vec{q}_c}{dt} = \frac{\partial \mathcal{H}}{\partial \vec{p}_c} \Rightarrow -\frac{d\vec{q}_r}{d\tilde{t}} = -\frac{\partial \mathcal{H}}{\partial \vec{p}_r} \checkmark$$

$$\frac{d\vec{q}_c}{dt}(t-\tilde{t}) = -\frac{d\vec{q}_c}{d\tilde{t}}(t-\tilde{t}) = -\frac{d\vec{q}_r}{d\tilde{t}}$$

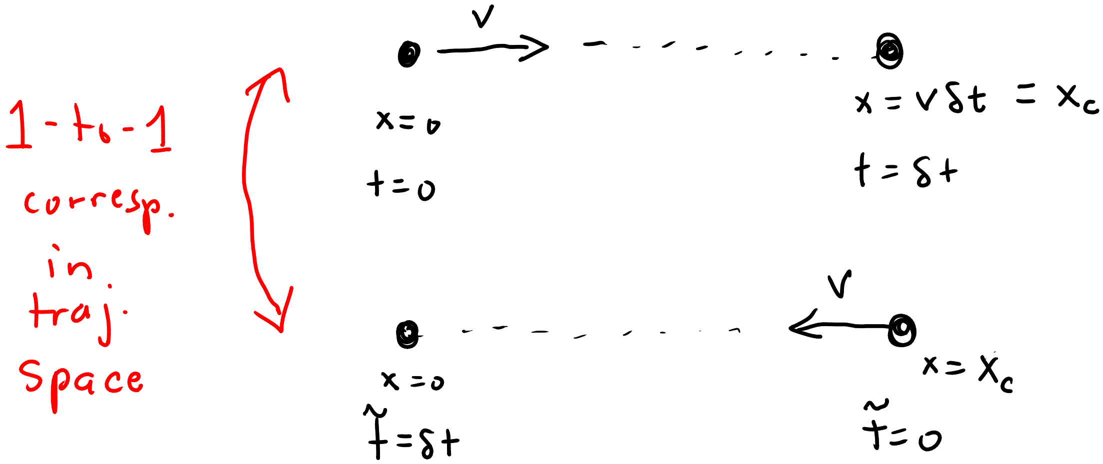
$$\frac{d\vec{p}_c}{dt} = -\frac{\partial \mathcal{H}}{\partial \vec{q}_c} \Rightarrow \frac{d\vec{p}_r}{d\tilde{t}} = -\frac{\partial \mathcal{H}}{\partial \vec{q}_r} \checkmark$$

apparent counterexample?

$$\vec{F} = q(\vec{v}\times\vec{B})$$

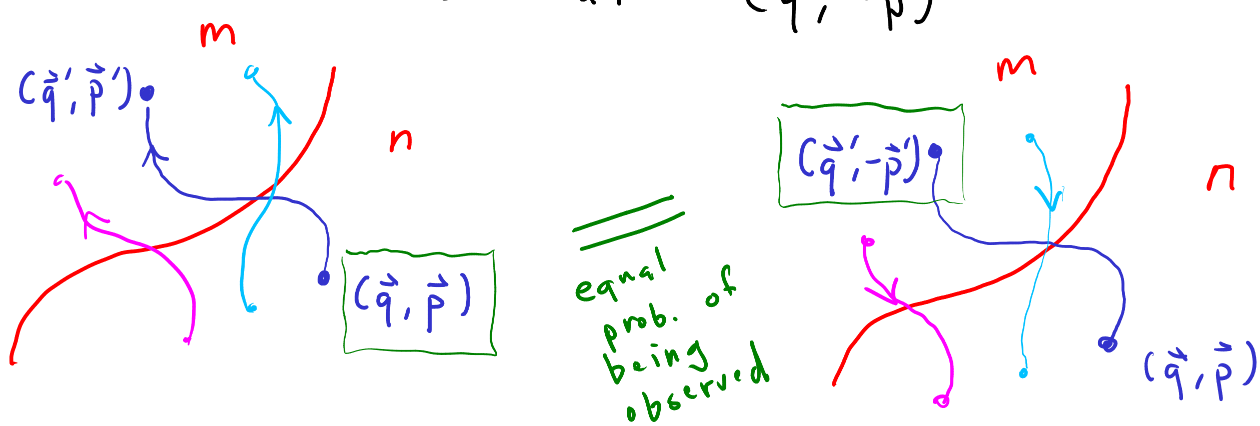


for $\vec{B}$ fields remember to apply time reversal to generator of $\vec{B}$ field ($\vec{B} \rightarrow -\vec{B}$)



argument: we approach the stationary as $t \to \infty$ & due to mixing all microstates equally likely

$t \to \infty$: • equally likely to be at $(\vec{q}, \vec{p})$ as at $(\vec{q}\,', -\vec{p}\,')$



$\Rightarrow$ equally likely to observe original & reversed trajectories

$$W_{\underset{n\to m}{mn}} = \frac{\#\text{ traj. start in } n \text{ \& end up in } m \text{ after } \delta t}{\#\ \text{"}\ \text{"}\ \text{"}\ \text{\& end up anywhere in } \delta t}$$

$\Rightarrow$ prob of observing $n \to m$ transition
$=$ prob. of observing $m \to n$ transition

$$\underbrace{\underset{\substack{\uparrow \\ \text{prob. } n\to m \\ \text{given a start} \\ \text{in } n}}{W_{mn}} \; \underset{\substack{\uparrow \\ \text{prob. of} \\ \text{starting} \\ \text{in } n}}{p_n^s}}_{\substack{\text{prob. of observing} \\ n\to m \text{ transition}}} = \underbrace{W_{nm}\, p_m^s}_{\substack{\text{prob. of} \\ \text{observing} \\ m\to n \text{ trans.}}}$$

local detailed balance (LDB)

$\Rightarrow$ tells us an important property of W matrix elements

even though LDB is derived using $t \to \infty$ limit, the matrix W describes dynamics at any time

$$\vec{p}(t+\delta t) = W \vec{p}(t)$$

$$W_{mn} = \mathcal{P}(m,t\,|\,n,0) \qquad W_{nm} = \mathcal{P}(n,t\,|\,m,0)$$

$$p_n^s = \mathcal{P}(n) \qquad p_m^s = \mathcal{P}(m)$$

Bayes: $$\mathcal{P}(n|m) = \frac{\mathcal{P}(m|n)\,\mathcal{P}(n)}{\mathcal{P}(m)}$$