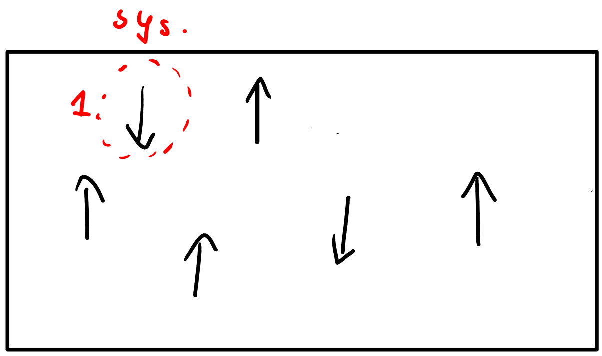


state 1 : spin 1 $\downarrow$

2 : spin 1 $\uparrow$

$$E_1 = 0$$
$$E_2 = \epsilon$$

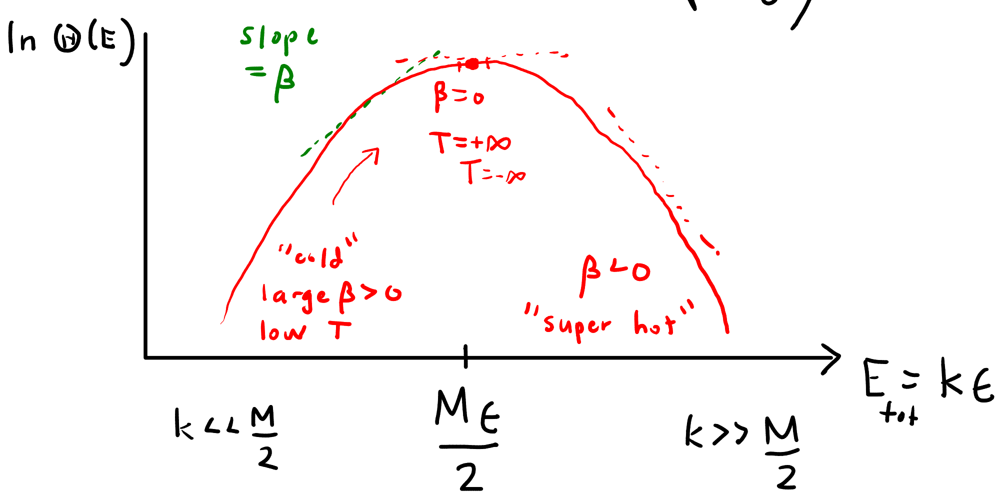
k up spins

$$E_{tot} = k\epsilon$$

$$\Theta_n = \Theta(E_{tot} - E_n)$$

$$\beta = \frac{1}{k_B T}$$

$$\Theta(E) = \binom{M}{E/\epsilon} \quad = \frac{\partial \ln \Theta}{\partial E}\Bigg|_{E_{tot}}$$



$(k = \frac{M}{2})$

Taylor approx:

$$\ln \Theta(E) \approx \text{const.} - \frac{\left(M - \frac{2E}{\epsilon}\right)^2}{M}$$

valid when

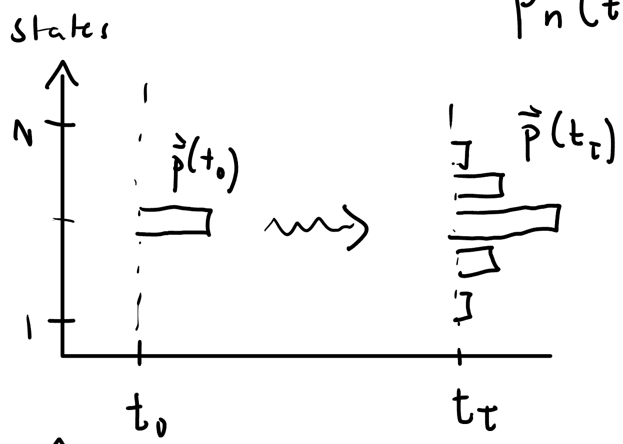
$$1 \ll k \ll N$$

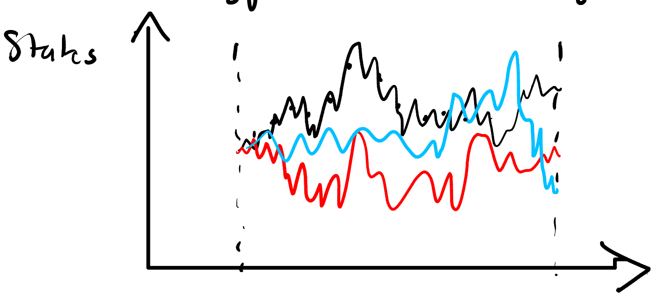
$$\Rightarrow \quad \beta = \frac{1}{k_B T} \cong \frac{2}{\epsilon}\left(1 - \frac{2E_{tot}}{\epsilon M}\right)$$

State picture: $\vec{p}(t_\tau) = W\,\vec{p}(t_{\tau-1})$

$$= W^\tau\,\vec{p}(t_0)$$

$p_n(t_\tau) \longrightarrow$ prob. to find state n at time t_τ





$\nu = (n_0, n_1, n_2, \ldots, n_\tau)$

$\mathcal{P}(\nu) =$ prob. of traj ν

$$= W_{n_\tau n_{\tau-1}} \cdots W_{n_2 n_1} W_{n_1 n_0}\, p_{n_0}(t_0)$$

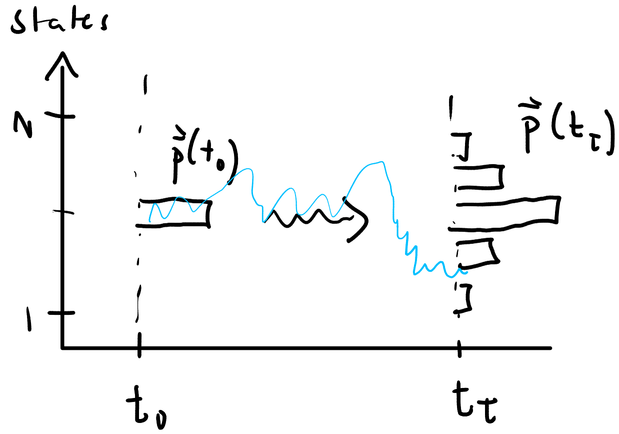
$$\sum_\nu \mathcal{P}(\nu) = \sum_{n_\tau} \cdots \sum_{n_0} \mathcal{P}(\nu) = 1$$

i) given a traj. ν, define "reverse" trajectory $\tilde{\nu}$ (reversing seq. of states, but still going forw. in time)

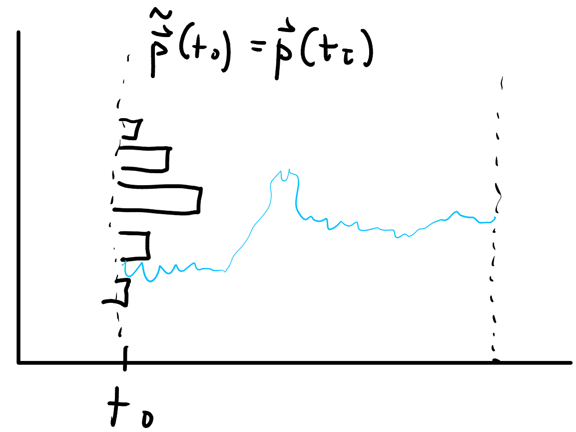
$$\nu = (n_0, n_1, n_2, \ldots, n_\tau)$$

$$\tilde{\nu} = (\underset{n_\tau}{\underset{\shortparallel}{\tilde{n}_0}}, \underset{n_{\tau-1}}{\underset{\shortparallel}{\tilde{n}_1}}, \tilde{n}_2, \ldots, \underset{n_0}{\underset{\shortparallel}{\tilde{n}_\tau}})$$

ii) reverse ensemble: initial distrib. of states is the <u>final</u> distrib. of our original ensemble "forward" ensemble



"reverse" ensemble



prob.

GOAL: compare ν in forw. ensemble : $\mathcal{P}(\nu)$

to $\tilde{\nu}$ in rev. ensemble : $\tilde{\mathcal{P}}(\tilde{\nu})$

rev. ensemble

rev. traj.

b/c W is ergodic + mixing $\Rightarrow$ LDB holds

MR: if $W_{nm} \neq 0 \Rightarrow W_{mn} \neq 0$

if $\mathcal{P}(\nu) \neq 0 \Rightarrow \tilde{\mathcal{P}}(\tilde{\nu}) \neq 0$

define ratio: "irreversibility"

$$I(\nu) \equiv k_B \ln \frac{\mathcal{P}(\nu)}{\tilde{\mathcal{P}}(\tilde{\nu})}$$

$|I(\nu)|$ is large when $\mathcal{P}(\nu) >> \tilde{\mathcal{P}}(\tilde{\nu})$

or $\tilde{\mathcal{P}}(\tilde{\nu}) >> \mathcal{P}(\nu)$

$I(\nu) = 0$ when $\mathcal{P}(\nu) = \tilde{\mathcal{P}}(\tilde{\nu})$

next time: $\left\langle e^{-I(v)/k_B T} \right\rangle = 1$