last time: special case system = total (isolated sys)

erg + mixing + $W = Q = 0$

2nd law: $I = \Delta S \geq 0$

$$\Delta S = S(t_\tau) - S(t_0)$$

valid for traj. of any length

$$S(t+\delta t) - S(t) \geq 0$$

$\Rightarrow$ $S(t)$ is non-decreasing

Do we know anything else?

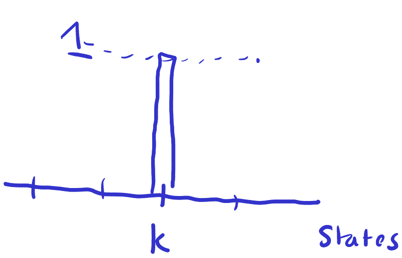
note: $S(t)$ is bounded

$$0 \leq S(t) \leq k_B \ln N$$

$$S(t) = -k_B \sum_{n=1}^{N} p_n(t) \ln \underbrace{p_n(t)}_{0 \leq \cdots \leq 1}$$

occurs:

$$p_n = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$$



$$1 \ln 1 = 0$$

$$0 \ln 0 = \lim_{x \to 0^+} x \ln x = 0$$

occurs:

$p_n = \frac{1}{N}$ for all n

$$-k_B \sum_n \frac{1}{N} \ln \frac{1}{N}$$

$$= -k_B \ln \frac{1}{N}$$

$$= k_B \ln N$$

proof of upper bound:

$$k_B \ln N - S(t) = k_B \ln N + k_B \sum_n p_n \ln p_n$$

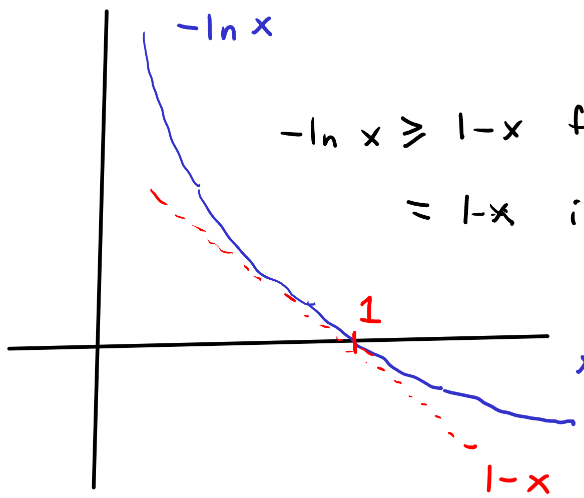
$$= k_B \sum_n p_n \ln N + k_B \sum_n p_n \ln p_n$$

$$= k_B \sum_n p_n \ln N p_n$$

$$= k_B \sum_n p_n \left(-\ln \frac{1}{N p_n}\right)$$

$$\geq k_B \sum_n p_n \left(1 - \frac{1}{N p_n}\right)$$

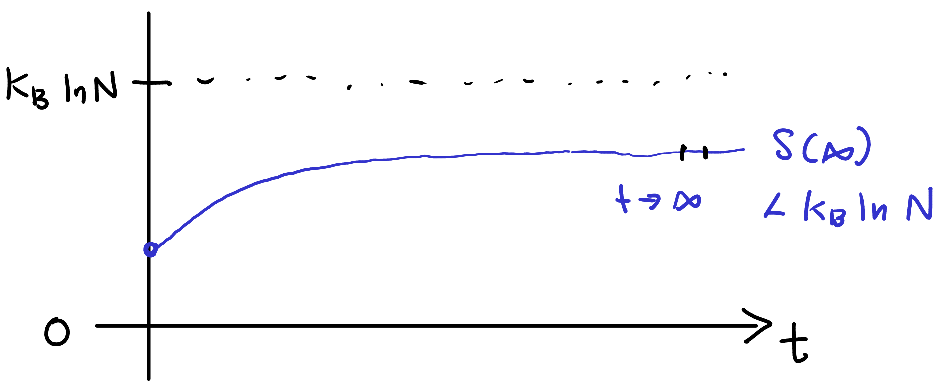
$$= k_B (1 - 1) = 0 \ \checkmark$$



$-\ln x \geq 1-x$ for all $x \geq 0$

$= 1-x$ if $x=1$

consider scenario:



is this possible?

ANS: no, due to thermo. laws

How to show this scenario is impossible?

Recall: IFT $\quad 1 = \langle e^{-I(\nu)/k_B} \rangle$

$$= \sum_\nu \mathcal{P}(\nu)\, e^{-I(\nu)/k_B}$$

$$\geq \sum_\nu \mathcal{P}(\nu) \left(1 - \frac{I(\nu)}{k_B}\right)$$

$$= 1 - \frac{1}{k_B} \langle I(\nu) \rangle$$

$e^{-x} \geq 1-x$

$= 1-x$ iff $x=0$

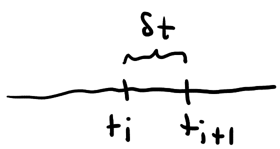
$$\Rightarrow \quad 1 \geq 1 - \frac{I}{k_B} \quad \Rightarrow \quad I \geq 0$$

if $I = 0$ only way this is possible is if $I(\nu) = 0$ for every possible ν

in the $t \to \infty$ limit: $\vec{p}(t) = \vec{p}^{\,s}$ station. prob.

look at traj. of length 1 in this limit:

$$\nu = \mu_i = (n_i, n_{i+1})$$



$S(t)$

$$0 = I(\mu_i)$$

$$= k_B \ln \frac{W_{n_{i+1} n_i} \overbrace{p_{n_i}(t_i)}^{p^s_{n_i}}}{W_{n_i n_{i+1}} \underbrace{p_{n_{i+1}}(t_{i+1})}_{p^s_{n_{i+1}}}}$$

$I = \Delta S = 0$

avg. $I = 0 \Rightarrow I(\mu_i) = 0$ for any μ_i

μ_i '' traj. of length 1

$$\Downarrow$$

$$W_{n_{i+1} n_i} p^s_{n_i} = W_{n_i n_{i+1}} p^s_{n_{i+1}}$$

also know: LDB

$$\frac{W_{n_{i+1} n_i}}{W_{n_i n_{i+1}}} = e^{-\beta(\underbrace{E_{n_{i+1}} - E_{n_i}}_{\substack{0 \\ \text{for} \\ \text{isol.} \\ \text{sys.}}} + \underbrace{\omega_{n_{i+1} n_i}}_{\substack{0 \\ \text{for} \\ \text{isol.} \\ \text{sys.}}})}$$

$$= 1$$

$\Rightarrow p^s_{n_i} = p^s_{n_{i+1}}$ for any n_i + n_{i+1} connected by trans.

$= \text{const.}$

$$\Rightarrow \quad p_{n_i}^s = \text{const.} \quad \text{for all } n_i$$

$\sum_{n_i} p_{n_i}^s = 1$

$$= \frac{1}{N}$$

by norm.

(b/c all states connected by paths in ergodic net)

$\Rightarrow$ for an isolated erg + mixing system stat. prob. $p_{n_i}^s = \frac{1}{N}$

$$\Rightarrow \quad S(t) = k_B \ln N \quad \text{as } t \to \infty$$

plateau has to be at Boltzmann value

$$S_{max} = k_B \ln N$$