case : 2) allow heat from env. $Q \neq 0$

but no coupling to work : $W = 0$

1st: $Q = \Delta E$

2nd: $T\,I = T\,\Delta S - \Delta E = -\Delta F \geq 0$

recall: $F = E - TS$

$\uparrow$ avg. energy $\quad \uparrow$ entropy

$\Delta F \leq 0$

for any length traj. ensemble

Helmholtz free energy is non-increasing

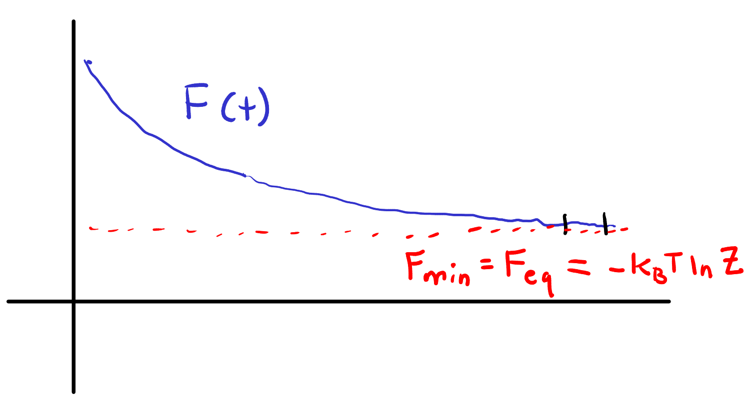
Similar argument: $F(t)$ cannot decrease forever

b/c $F(t) = E(t) - T\,S(t)$

$S(t)$: $0 \leq S(t) \leq k_B \ln N$

$E(t) = \sum_n p_n(t) E_n$: $E_{min} \leq E(t) \leq E_{max}$

smallest sys. state energy $\quad$ largest " "



look at ensemble of trajectories at long times (plateau) :

$\Delta F \approx 0 \Rightarrow I = 0$

$$I = 0 \Rightarrow \underset{(n_i, n_{i+1})}{\overset{\shortparallel}{I(\mu_i)}} = 0 = k_B \ln \frac{W_{n_{i+1} n_i} p_{n_i}(t_i)}{W_{n_i n_{i+1}} p_{n_{i+1}}(t_{i+1})}$$

$$\frac{W_{n_{i+1} n_i}}{W_{n_i n_{i+1}}} = e^{-\beta(E_{n_{i+1}} - E_{n_i} + \cancel{\omega_{n_{i+1} n_i}}^{\,0})}$$

numerator = denominator

$$e^{\beta E_{n_i}} \underbrace{p_{n_i}(t_i)}_{p^s_{n_i}} = e^{\beta E_{n_{i+1}}} \underbrace{p_{n_{i+1}}(t_{i+1})}_{p^s_{n_{i+1}}}$$

$$Z = \sum_{n=1}^{N} e^{-\beta E_n}$$

$\Rightarrow$ satisfied when: $p_n^s = \dfrac{e^{-\beta E_n}}{Z}$

as $t \to \infty$ $\quad p_n(t) \longrightarrow \dfrac{e^{-\beta E_n}}{Z}$ Boltz. equil. distrib.

$$F(t) = E(t) - TS(t)$$

plug in p_n^s $\Rightarrow$ $F_{eq} = -k_B T \ln Z$

$$E(t \to \infty) = \sum_n p_n^s E_n = \frac{1}{Z} \sum_n e^{-\beta E_n} E_n$$

$$S(t \to \infty) = -k_B \sum_n p_n^s \ln p_n^s$$

$$= -k_B \sum_n \frac{e^{-\beta E_n}}{Z} (-\beta E_n - \ln Z)$$

$$\Rightarrow F(t \to \infty) = -k_B T \ln Z$$

Interpretation of entropy as information:
Shannon source coding theorem

surprisal: $-k_B \ln p_n(t)$

entropy: avg. surprisal $S(t) = -k_B \sum_n p_n(t) \ln p_n(t)$

imagine scenario: Alice (A) is a prof.
Bob (B) is a grad. student

experiment: system (stochastic) $N = 4$ states

one run:

repeat many times
$\left\{ \right.$
- B prepares sys. in init. dist. $\vec{p}(0)$
- B lets it evolve for time t
- B takes measurement, records state

output: series of state measurements
i.e. 2, 4, 1, 3, 2, ...
← results of separate runs

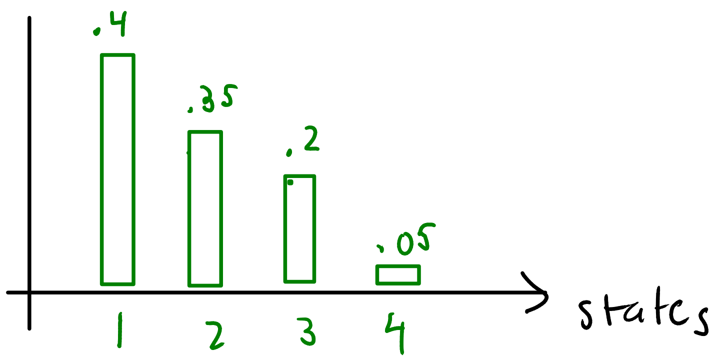
A + B know: sys. trans. matrix W
prob. $\vec{p}(0)$
hence $\vec{p}(t) = W^t \vec{p}(0)$

A doesn't know seq. of measurements

GOAL: Bob has to encode seq. + send it w/ fewest bits possible

$\vec{p}(t)$

distrib. of states at end of each run



case I: Bob is lazy

state	code
1	00
2	01
3	10
4	11

easy to read (every 2 bits is a state)

if there were N states in general

mean message length per state $= \log_2 N$