reach $B = B_{min}$ (optimal coding)

when $q_n = p_n$ for all n

$$2^{-\ell_n} = p_n$$

$$\Rightarrow \ell_n = -\log_2 p_n$$ optimal code length for each state n

however you are restricted to integer ℓ_n

optimality among real codes > Shannon prediction (Huffman coding)

Reminder: assumptions

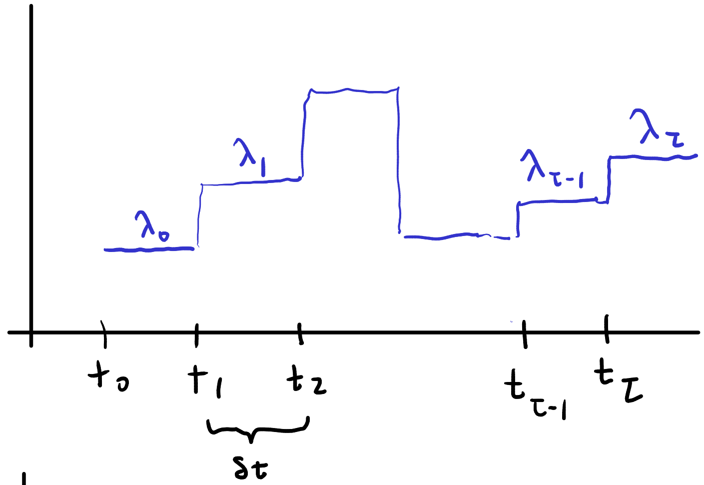
A) environment constant $\Rightarrow W_{nm}$ constant

B) single environment w/ some temp. T

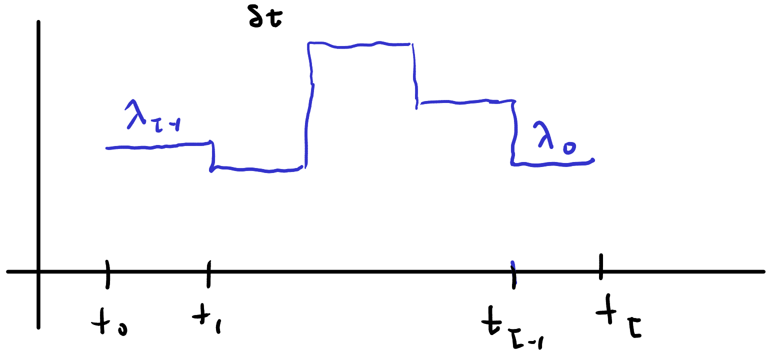
Relax assumption: allow $W_{nm}(\lambda(t))$

$\lambda(t)$ = set of external parameters we <u>control</u>: temp., pressure, etc.

Control protocol: $\lambda(t)$ known beforehand forward control protocol $\lambda(t)$



reverse protocol $\tilde{\lambda}(t)$



generalized irreversibility:

$$I(\nu) = k_B \ln \frac{\mathcal{P}(\nu)}{\tilde{\mathcal{P}}(\tilde{\nu})}$$

prob. of traj. ν under forw. protocol in forw. ensemble

prob. of rev. traj. $\tilde{\nu}$ under rev. protocol in rev. ensemble

$\nu = (n_0, n_1, n_2, \ldots, n_\tau)$

$$= k_B \ln \frac{W_{n_\tau n_{\tau-1}}(\lambda_{\tau-1}) \cdots\cdots W_{n_2 n_1}(\lambda_1) W_{n_1 n_0}(\lambda_0)\, p_{n_0}(t_0)}{W_{n_0 n_1}(\lambda_0) \cdots\cdots W_{n_{\tau-1} n_\tau}(\lambda_{\tau-1})\, p_{n_\tau}(t_\tau)}$$

$$= \sum_{i=0}^{\tau-1} I(\mu_i) \qquad \mu_i = (n_i, n_{i+1})$$

$$I(\mu_i) = k_B \ln \frac{W_{n_{i+1} n_i}(\lambda_i)}{W_{n_i n_{i+1}}(\lambda_i)} \frac{p_{n_i}(t_i)}{p_{n_{i+1}}(t_{i+1})}$$

every proof from before is same:

$$\text{IFT} \quad \langle e^{-I(\nu)/k_B} \rangle = 1$$

$$\rightarrow \quad I = \langle I(\nu) \rangle \geq 0$$

$$= 0 \quad \text{iff } \mathcal{P}(\nu) = \tilde{\mathcal{P}}(\tilde{\nu})$$

1st: $Q = \Delta E + \overline{W}$

2nd: $I = -\dfrac{\Delta F}{T} - \dfrac{\overline{W}}{T} \geq 0 \qquad F = E - TS \qquad$ etc.

Special case: $\lambda(t)$ is periodic

$$\lambda(t+\tau) = \lambda(t) \qquad \text{period } \tau$$

$$W(\lambda(t)) = W(\lambda(t+\tau))$$

PS #2 proof: $t \to \infty$ you approach periodic state

$$p_n(t) = p_n(t+\tau)$$

$E(t) = \sum_n p_n(t) E_n$ is also periodic

$S(t) = -k_B \sum_n p_n(t) \ln p_n(t)$ " "

$\Delta E = E(\tau) - E(0)$ over one period

$= 0$

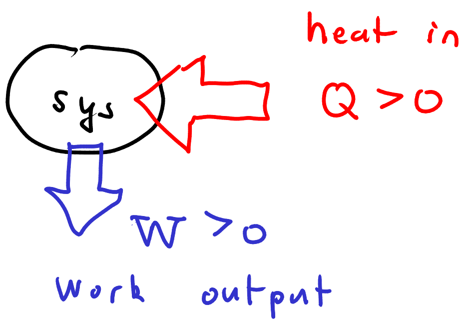
$\Delta F = F(\tau) - F(0) = 0$

1st: $Q = \bar{W}$

2nd: $\bar{W} = -T\mathcal{I}$

$\mathcal{I} \geq 0$

$$Q = \bar{W} = -T\mathcal{I} \leq 0 \quad \text{for positive } T$$



not allowed!

Kelvin-Planck version of 2nd law:

you cannot create a cyclically driven system connected to env. at one temp. T that produces net work in long term