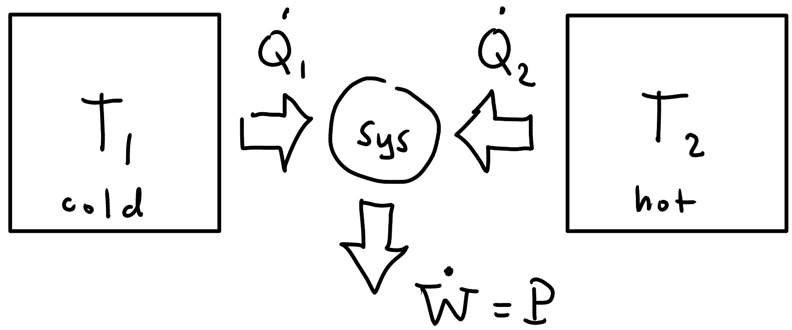

$$\sigma = \frac{I}{\delta t}$$

$$T_2 \geq T_1$$

1st: $\dot{Q}_1 + \dot{Q}_2 = P$

2nd: $\sigma = -\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2} \geq 0$

cycle or stat. state

case 1: $\dot{Q}_2 > 0 \Rightarrow \dot{Q}_1 < 0$

$$\Rightarrow P \leq \dot{Q}_2 \left(1 - \frac{T_1}{T_2}\right)$$

$$\eta = \frac{P}{\dot{Q}_2} \leq 1 - \frac{T_1}{T_2}$$ Carnot bound

note: 1) if $T_1 = T_2 \Rightarrow$ same temp. for both env's

$\Rightarrow \eta \leq 0$ no positive net power output is possible in long term

(Kelvin-Planck again)

$$P = P_{out} - P_{in}$$

$$\Rightarrow P \leq 0$$

$$\Rightarrow P_{out} \leq P_{in}$$

2) technically T_1 + T_2 will change over long times as heat / flows in / out but we assume env >> sys so these changes are negligible

3) when do we reach $\eta = \eta_{max} = 1 - \frac{T_1}{T_2}$?

$$\sigma = \frac{I}{\Delta t} \to 0$$

two options: • if we are in stat. state

$\Delta t = \delta t$

so $I \to 0$ equilibrium

$T_1 = T_2$ + let sys. equilibrate

$\eta = \eta_{max} = 1 - \frac{T_1}{T_2} = 0$ boring

• if we are cyclically driving:

$\Delta t = \tau \, \delta t$ cycle periodic

$\Delta t \to \infty$ (∞ long cycle)

$T_1 \neq T_2$ $\quad I \neq 0 \Rightarrow W = Q_1 + Q_2 > 0$

power $\frac{W}{\Delta t} = P \to 0$ achieve $\eta = \eta_{max} > 0$ but zero power output

case 2: $\dot{Q}_1 > 0$ (draw heat out of cold env.)

$\Rightarrow$ refrigerator or heat pump

1st: $P = \dot{Q}_1 + \dot{Q}_2$

2nd: $\sigma = -\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2} \geq 0$ $\rightarrow$ $\frac{\dot{Q}_2}{T_2} = -\frac{\dot{Q}_1}{T_1} - \underbrace{\sigma}_{\geq 0}$

$\Rightarrow \dot{Q}_2 < 0$

dumping heat into hot env.

$$\dot{Q}_2 = -\frac{T_2}{T_1}\dot{Q}_1 - T_2\sigma \qquad \frac{T_2}{T_1} > 1$$

$$|\dot{Q}_2| > |\dot{Q}_1|$$

$$\Rightarrow P = \dot{Q}_1 + \dot{Q}_2 = \dot{Q}_1 - |\dot{Q}_2| < 0$$

$$P_{out} < P_{in}$$

running sys. requires net power input

plug in fridge!

coeff. of performance $\eta_R = \frac{\dot{Q}_1}{-P} = \frac{\text{heat rate out of cold bath}}{\text{input power}}$

$$\eta_R = \frac{\dot{Q}_1}{-\dot{Q}_1 - \dot{Q}_2} = \underset{\text{algebra}}{\cdots} = \frac{T_1}{T_2 - T_1} - \underbrace{\frac{T_1 T_2 \sigma}{T_2 - T_1}}_{\geq 0}$$

$$\Rightarrow \eta_R \leq \underbrace{\frac{T_1^{\sim 280\,K}}{\underbrace{T_2 - T_1}_{20\,K}}}_{\eta_R^{max}}$$

in general $\eta_R^{max} \sim O(10)$ for most practical cases

Big picture: "many children of IFT"

- IFT
  - 2nd law consequences (max. S or min. F) const. env., one temp
  - Jarzynski equality (varying env., one temp.)
  - bounds on heat engines/pumps (multiple temps)
  - systems near equilibrium (small I)
    - linear thermodynamics
      - Onsager reciprocity
      - fluctuation-dissipation theorem

Linear thermodynamics:

single temp. T + stationary state (not necessarily equil.)

2nd: $I = -\frac{1}{T}\cancelto{0}{\Delta F} - \frac{1}{T} W \geq 0$

1st: $Q = \cancelto{0}{\Delta E} + W$

$$\Rightarrow \quad I = -\frac{1}{T} W = -\frac{1}{T} Q \quad \Rightarrow \quad \underbrace{Q}_{\substack{\leq 0 \\ \text{heat} \\ \text{into env.}}} = \underbrace{W}_{\substack{\leq 0 \\ \text{work} \\ \text{on} \\ \text{sys.}}} = -TI \leq 0$$

$I = 0 \Rightarrow$ ESS $(Q = W = 0)$

$I > 0 \Rightarrow$ NESS $(Q = W < 0)$

near equilibrium: I small

($Q = \dot{W}$ is also small)