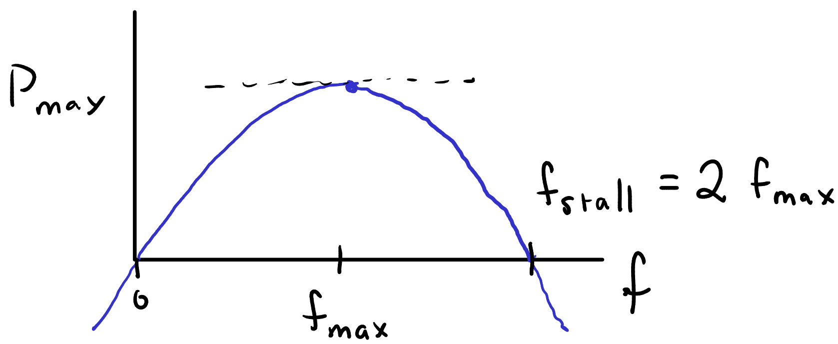
$$\underset{\text{net power output}}{P} = f\dot{x} = -f\left(L_{WH}\,\phi_H + L_{WW}\,\phi_W\right)$$

$$= -\frac{L_{WW}}{T}\left(f + \frac{L_{WH}\,\Delta T}{2L_{WW}T}\right)^2 + \frac{L_{WH}^2\,\Delta T^2}{4L_{WW}T^3}$$

case 1: $L_{WH} = L_{HW} < 0$



$$= -\frac{L_{WH}\,\Delta T}{2L_{WW}T}$$

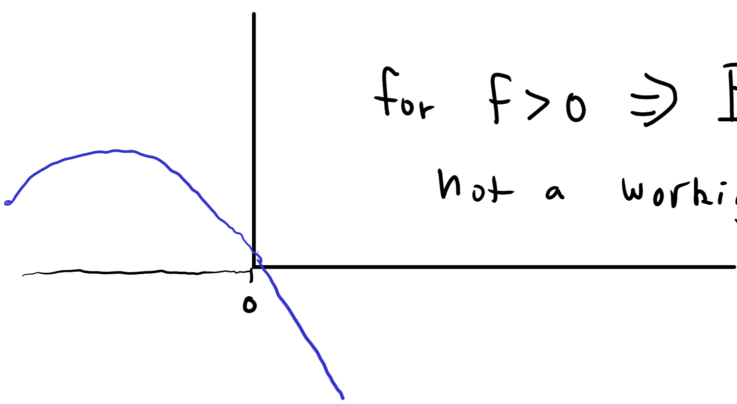
at $f = f_{max}$

$$P = P_{max} = \frac{L_{WH}^2\,\Delta T^2}{4L_{WW}T^3}$$

at $f = f_{stall}$

$$P = 0$$

case 2: $L_{WH} = L_{HW} > 0$



for $f > 0 \Rightarrow P < 0$

not a working engine

focus on case 1:

$$\dot{Q}_2 = J_H = L_{HH}\,\phi_H + L_{HW}\,\phi_W$$

$$= L_{HH}\frac{\Delta T}{T^2} + L_{HW}\frac{f}{T}$$

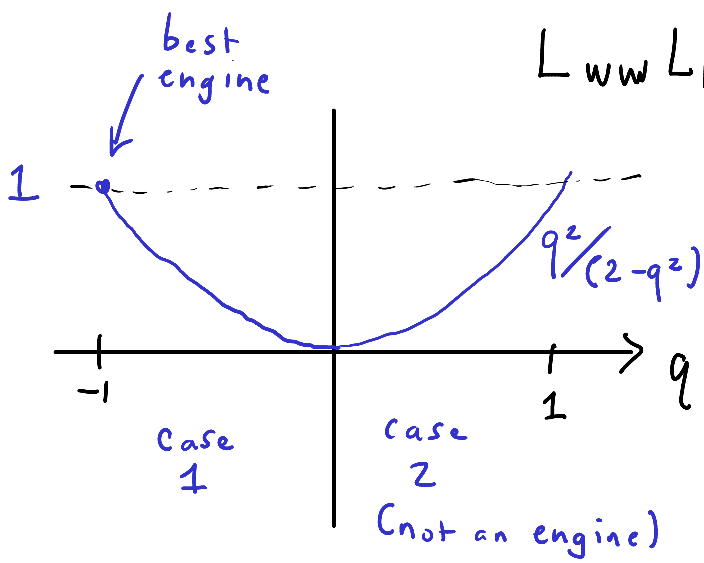
at $f = f_{max}$: $$= \frac{\Delta T}{T^2}\left(L_{HH} - \frac{L_{WW}L_{WH}}{2L_{WW}}\right)$$

$$\eta = \frac{P_{max}}{\dot{Q}_2(f=f_{max})} = \frac{\Delta T}{2T}\,\frac{q^2}{2-q^2} \qquad q \equiv \frac{L_{WH}}{\sqrt{L_{WW}L_{HH}}}$$

note: $\det(L) \geq 0$

$$L_{WW}L_{HH} - L_{WH}^2 \geq 0 \Rightarrow q^2 \leq 1$$

$$-1 \leq q \leq 1$$



for best engine $q = -1$

$$\Rightarrow \eta_{best} = \frac{\Delta T}{2T}$$

efficiency at max. power $\quad \eta \leq \dfrac{\Delta T}{2T}$

(for small ΔT + f values)

Carnot efficiency: $$\eta_c = 1 - \frac{T_1}{T_2}$$
$$= 1 - \frac{T}{T+\Delta T}$$

$$\approx \frac{\Delta T}{T}$$

at max. power $\quad \eta \lesssim \frac{1}{2}\eta_c$

Quantum stat. mech.

ensemble: many copies of system prepared at $t=0$

- classical: $p_n(0) =$ frac. of ensemble in classical state n at $t=0$

- quantum: $p_n(0) =$ " " " in quantum state $|\psi_n\rangle$ at $t=0$

$\{|\psi_n\rangle\}$ some arbitrary set of states in a Hilbert space, not necessarily orthog. or complete

but we require $\langle\psi_n|\psi_n\rangle = 1$ (norm.)

example: N_{tot} copies

ensemble: $\underbrace{|\psi_1\rangle \; |\psi_1\rangle \; |\psi_1\rangle}_{p_1(0) = \frac{3}{N_{tot}}} \; \underbrace{|\psi_2\rangle \; |\psi_2\rangle}_{p_2(0) = \frac{2}{N_{tot}}} \; |\psi_3\rangle \; \ldots \quad \ldots$

$p_n(0) \geq 0 \quad \forall n \qquad \sum_n p_n(0) = 1$

in classical case we had avg. over ensemble, i.e.

$$E = \langle E \rangle = \sum_n p_n E_n \quad , \text{ etc.}$$

in quantum case, each observable A has an operator $\hat{A}$

avg. of A in state $|\psi_n\rangle$: $\langle \psi_n | \hat{A} | \psi_n \rangle$

ensemble avg: $\langle A \rangle = \sum_n p_n \langle \psi_n | \hat{A} | \psi_n \rangle$

$\hat{A}$ is Hermitian: e-vecs $|a\rangle$

$\sum_a |a\rangle\langle a| = \underbrace{\hat{I}}_{\text{identity}} \qquad \hat{A}|a\rangle = \underbrace{a}_{\text{e-val}} |a\rangle$

$$\langle A \rangle = \sum_{n,a} p_n \langle \psi_n | \hat{A} | a \rangle \langle a | \psi_n \rangle \quad (1)$$

$$= \sum_{n,a} p_n \, a \langle \psi_n | a \rangle \langle a | \psi_n \rangle$$

$$= \sum_{n,a} a \underbrace{p_n |\langle a | \psi_n \rangle|^2}_{\text{two contributions to prob:}}$$

- p_n : prob to find $|\psi_n\rangle$ in ensemble
- $|\langle a|\psi_n\rangle|^2$: prob. to find result a when measuring $\hat{A}$ in state $|\psi_n\rangle$

rewrite Eq. 1 : $\langle A\rangle = \sum_{n,a} p_n \langle a|\psi_n\rangle\langle\psi_n|\hat{A}|a\rangle$

$$= \sum_a \langle a| \underbrace{\left[\sum_n p_n |\psi_n\rangle\langle\psi_n|\right]}_{\equiv\, \hat{\rho}} \hat{A}\, |a\rangle$$

$\equiv \hat{\rho}$ density operator for ensemble

$$\hat{\rho} = \sum_n p_n |\psi_n\rangle\langle\psi_n|$$

$$\langle A\rangle = \sum_a \langle a|\hat{\rho}\hat{A}|a\rangle$$

$$= \mathrm{tr}\,(\hat{\rho}\hat{A})$$

$$\hat{\rho}\hat{A} = \begin{pmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \end{pmatrix}$$

(actually basis indep.)