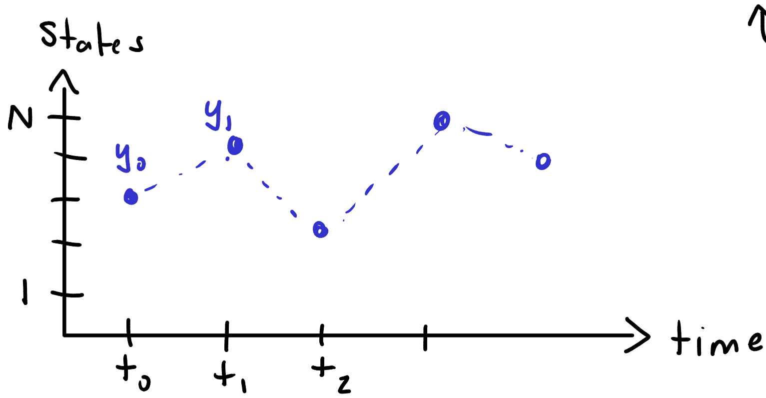
system variable $y(t)$ = labels macrostate of system at time t (could be vector)

discretize time $t_i = i\,\delta t \qquad i = 0, 1, 2, \ldots.$

$y(t_i) \equiv y_i$ discretize state

$$1 \le y_i \le N$$

N ← # macrostates

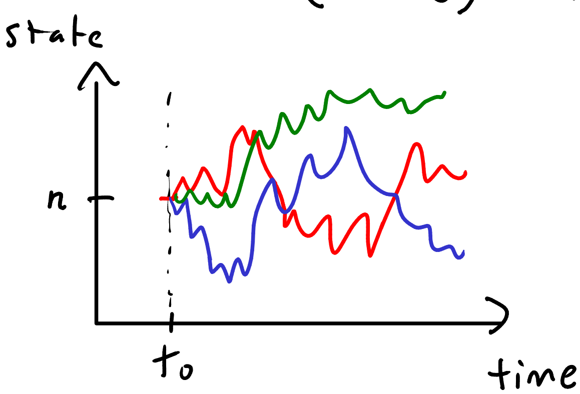


trajectory

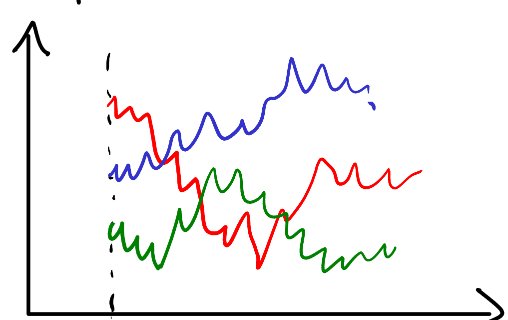
$\nu = (y_0, y_1, y_2, \ldots, y_i)$

= seq. of states associated w/ single exper. run

ensemble = collection of traj. from many ($\to \infty$) indep. exper. runs



pure ensemble



mixed ensemble

prob. dist. of initial state y_0

$$P(y_0) = \delta_{y_0, n} = \begin{cases} 1 & \text{if } y_0 = n \\ 0 & \text{if } y_0 \neq n \end{cases}$$

$P(y_0)$ <u>not</u> a delta func.

$$\mathcal{P}(\nu) = \mathcal{P}(y_0, y_1, y_2, \ldots, y_i)$$

$$\equiv \frac{\#\ \text{traj. w/ seq. } (y_0, y_1, \ldots, y_i) \text{ in ensemble}}{N_{traj} = \text{tot. \# of traj. in ensemble}}$$

$$\sum_{\nu} \mathcal{P}(\nu) = \sum_{y_0=1}^{N} \sum_{y_1=1}^{N} \cdots \sum_{y_i=1}^{N} \mathcal{P}(y_0, y_1, \ldots, y_i) = 1$$

physical quantity associated w/ traj. ν

$Q(\nu)$

ensemble avg. $\langle Q \rangle = \sum_{\nu} Q(\nu) \mathcal{P}(\nu)$

example: $Q(\nu) = E_{y_i} - E_{y_0}$ = total change in energy

$\langle Q \rangle$ = avg. energy change

Review of prob. concepts:

- underlying ensemble
- let A + B are "events" in this ensemble

example: $A = y_3$ state at time t_3

or $A = (y_0, y_1, y_2)$

$$\mathcal{P}(A) = \frac{\#\text{ traj. where } A \text{ occurs}}{N_{traj}}$$

joint prob $\quad \mathcal{P}(A,B) = \dfrac{\#\text{ traj. where } A \text{ \underline{and} } B \text{ occur}}{N_{traj}}$

marginal prob. $\quad \mathcal{P}(A) = \sum_B \mathcal{P}(A,B)$

$$\mathcal{P}(B) = \sum_A \mathcal{P}(A,B)$$

all prob's sum to 1: $\quad \sum_{A,B} \mathcal{P}(A,B) = 1$

$$\sum_A \mathcal{P}(A) = 1 \qquad \sum_B \mathcal{P}(B) = 1$$

conditional prob: $\quad$ prob. of A given that B occurs

$$\mathcal{P}(A|B) = \frac{\#\text{ traj. where } A \text{ \underline{and} } B \text{ occur}}{\#\text{ traj. where } B \text{ occurs}}$$

$$= \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)}$$

note: $\quad \sum_A \mathcal{P}(A|B) = \sum_A \dfrac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \dfrac{1}{\mathcal{P}(B)} \overbrace{\sum_A \mathcal{P}(A,B)}^{\mathcal{P}(B)}$

$$= 1$$

but $$\sum_B P(A \mid B) = \sum_B \frac{P(A,B)}{P(B)} \neq 1$$ in general

A & B can be in the past or future relative to each other

time	t_0	t_1	t_2	...	t_4
	1	1	2	1	3
•	1	4	3	2	1
	1	1	3	1	4
		...			

•
•

$A =$ state at t_1
$= 4$

$B =$ state at t_3
$= 2$

$P(A \mid B)$ $P(B \mid A)$

if A & B are <u>independent</u>:

$$P(A,B) = P(A)P(B)$$

$$P(A \mid B) = \frac{P(A,B)}{P(B)} = P(A)$$

$$P(B \mid A) = \frac{P(A,B)}{P(A)} = P(B)$$

Bayes theorem $$\boxed{P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}}$$

prove: $\mathcal{P}(B|A) = \dfrac{\mathcal{P}(A,B)}{\mathcal{P}(A)}$ plug into RHS

$$\Rightarrow \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \mathcal{P}(A|B) \quad \text{LHS}$$

example: 1) Statement: I have 3 kids + at least one is a boy.

What is the prob. that I have 3 boys?

$\tilde{B}$ = at least 1 of my kids is a boy

BBB = all three are boys

$$\mathcal{P}(BBB \mid \tilde{B}) = ?$$

all possible traj.

$$8\left\{\begin{array}{l} \left.\begin{array}{l} BBB \\ BBG \\ BGB \\ \cdots \end{array}\right\} \\ GGG \end{array}\right.$$

$\tilde{B}$ is true for 7 out of 8

$$\mathcal{P}(BBB \mid \tilde{B}) = \frac{1}{7}$$

$\underbrace{1}$ $\underbrace{1/8}$

check Bayes: $\mathcal{P}(BBB \mid \tilde{B}) = \dfrac{\mathcal{P}(\tilde{B} \mid BBB)\,\mathcal{P}(BBB)}{\underbrace{\mathcal{P}(\tilde{B})}_{7/8}}$

$= \dfrac{1}{7}$

2) I have 3 kids. Each of them rolled a pair of dice. I have at least one boy who rolled (1, 1). What is the prob. I have 3 boys?