How to get LDB? $\quad \dfrac{W_{ij}}{W_{ji}} = e^{-\beta(E_i - E_j)}$

$$\Downarrow \qquad \beta = \frac{1}{k_B T}$$

Boltzmann equil.

$$p_i^s = \frac{e^{-\beta E_i}}{Z}$$

typical assumption: for a quantum sys. connected to an env. at temp T

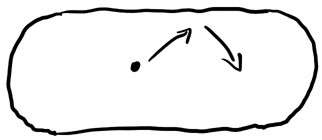
$$\hat{\rho}(0) = \begin{pmatrix} \diagdown & \wr \\ \wr & \diagdown \end{pmatrix} \xrightarrow[t \to \infty]{} \hat{\rho}^s = \begin{pmatrix} \diagdown & 0 \\ 0 & \diagdown \end{pmatrix}$$

E_i = energy eigenvalues $\qquad \rho_{ii}^s = \dfrac{e^{-\beta E_i}}{Z}$

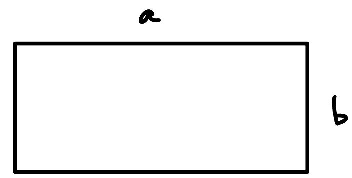
systems	classical		quantum
chaotic:	ergodicity, mixing	$\Rightarrow$	?
not chaotic:	integrable; if you have 2N dim. phase space $(\vec{q}, \vec{p})$	$\Rightarrow$	?

you have N conserved quantities (max.)

billiards:



Bunimovich: chaotic

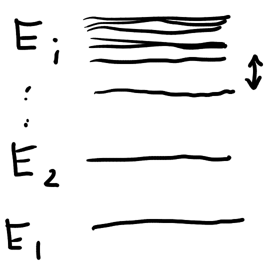


rectangular: not chaotic

in QM these are 2D particle in a box systems $\Rightarrow$ easy to solve (numerically)

"integrability" in a quantum context: want degenerate (or nearly deg.) energy levels

focus on spacings b/t consecutively ordered energy levels:



normalized spacing:

$$s_i = \frac{E_{i+1} - E_i}{\langle E_{i+1} - E_i \rangle}$$

avg. over all spacings

rectangular box

a/b is irrational $\Rightarrow$ no trivial degeneracies

$$E_{n,m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi n}{a}\right)^2 + \left(\frac{\pi m}{b}\right)^2\right]$$

n, m integers

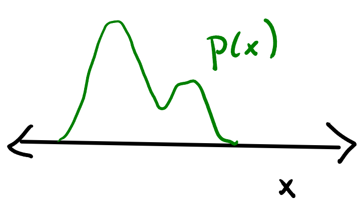
$i = (n,m)$ order E_i from smallest to largest

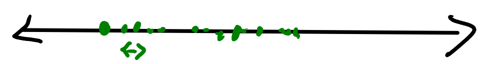
1977: Berry-Tabor conjecture:

completely integrable (hence not chaotic) systems generically will have quantum analogues where dist. of normalized spacings s looks like: $P(s) = e^{-s}$

few special counter-examples

imagine 1D prob. dist. $p(x)$ & I draw many x from this distrib. independently





↳ normalized spacings
$P(s) = e^{-s}$

$\Rightarrow$ energy levels in an integrable behave as if they are independently drawn random variables

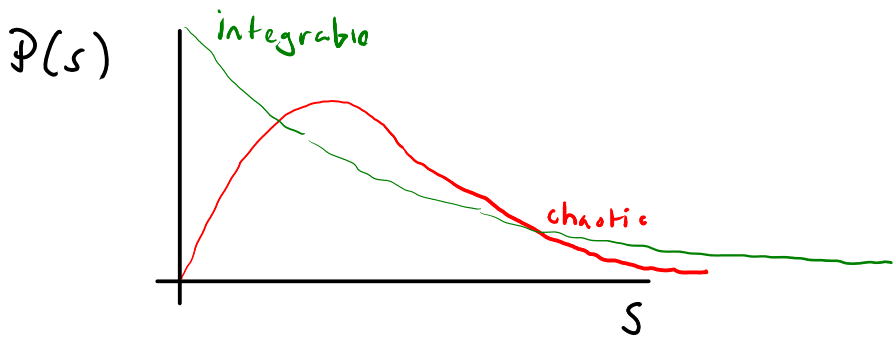
any deviation from $P(s) = e^{-s} \Rightarrow$ has to be correlations (i.e. "repulsion", etc.)

b/t energy levels

1984: BGS (Bohigas Giannoni Schmit)

conjecture: quantum version of a chaotic sys will generically have $\mathcal{P}(s) = \alpha s^b e^{-\beta s^2}$ $\quad \alpha, \beta \geq 0$, $b \geq 1$

note: $\mathcal{P}(s=0) = 0$



If I generically choose a Hermitian matrix H w/ elements drawn from a real Gaussian distrib.

$\Rightarrow$ random matrix theory $\quad \mathcal{P}(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4} s^2}$ $\quad$ Wigner distrib.