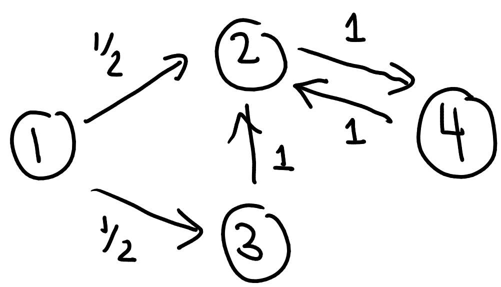
for an ergodic network: $p_i^s = \frac{1}{h_{ii}}$

avg. time to make trip from $i \to i$

example of using stat. prob's + MHT in practice: Google Page Rank

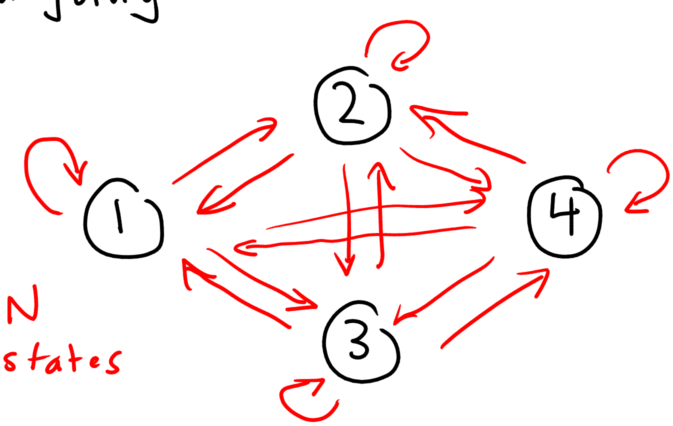
$\rightarrow$ = hyperlink

(i) = webpage



trans. matrix W^H

W^H = "transition" matrix
created by assigning equal prob's to all outgoing arrows



"complete" network: all states connected to all states

trans. prob.

$$W^C_{ij} = \frac{1}{N} \text{ for all } i,j$$

$j \to i$

$W = \alpha W^H + \beta W^C$

$\alpha + \beta = 1$

$\Rightarrow$ columns of W sum to 1

$\Rightarrow$ proper trans. matrix W

$\beta > 0$: ergodic network

$\alpha = 0.85$

$\beta = 0.15$

$\Rightarrow$ Calculate $p_j^{ss} = \frac{1}{h_{jj}}$ & use it to rank webpages

<u>not</u> using N^2 equ's for h_{ij} to solve this

better approach:

start w/ arbitrary $\vec{p}^{\,0}$

$W \cdots\cdots W \vec{p}^{\,0} \approx \vec{p}^{\,s}$

final puzzle piece: show $W^n \vec{p}^{\,0} \to \vec{p}^{\,s}$ as $n \to \infty$ for any starting $\vec{p}^{\,0}$

Next step: develop a physical framework

classical mech. for "complex" systems $\Longrightarrow$ W matrices

$\swarrow$ MR

$\searrow$ $W^n \vec{p}^{\,0} \overset{n\to\infty}{=} \vec{p}^{\,s}$

Mechanics in d-dim. w/ M particles:

$\vec{q}$ = dM coordinates for all particl. } 2dM
$\vec{p}$ = dM momenta " " " } dim. phase space

exact state of system $\Longleftrightarrow$ $(\vec{q}, \vec{p})$ in phase space

assumptions: system has finite volume V
no gain / loss of energy from outside (closed)

total energy is conserved

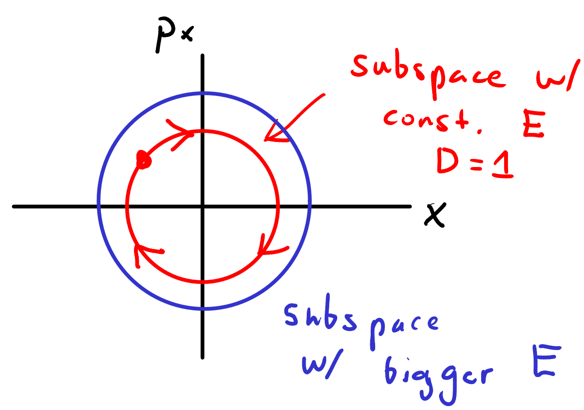
$E = \mathcal{H}(\vec{q}, \vec{p})$ Hamiltonian
↑
defines a subspace in phase space w/ dim. $2dM - 1 \equiv D$

1-dim. harm. oscill.

$$\mathcal{H} = \frac{1}{2} p_x^2 + \frac{1}{2} x^2 = E$$

phase space: (x, p_x)

$d = 1$, $M = 1$ $2dM = 2$

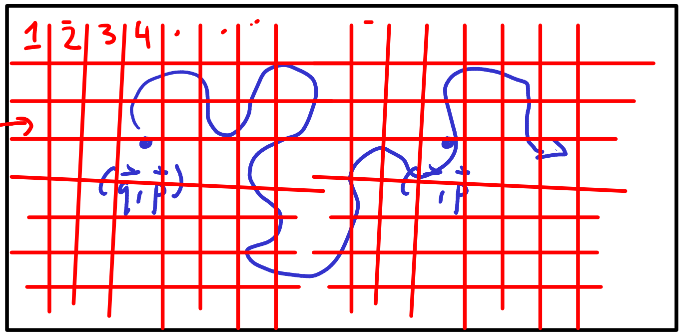


focus: a single subspace (one layer of onion)

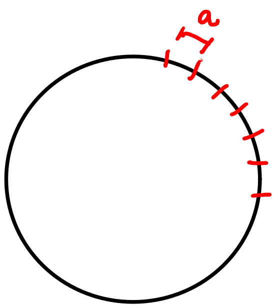
$= D$-dim. surface

cartoon:

box: "microstate"



divide up surface into D-dim. boxes of volume a



radius $= \sqrt{2E}$

circumf. $2\pi\sqrt{2E}$

$$\Theta(E) = \frac{2\pi\sqrt{2E}}{a}$$

label boxes:

$$\mu = 1, 2, 3, \ldots, \frac{\Theta(E)}{a}$$

typically leave out a in our notation